THE

MATHEMATICAL GAZETTE.

EDITED BY

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WITH THE CO-OPERATION OF

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PRELIMINARY REPORT OF SCHOLARSHIPS SUB-COMMITTEE.

[NOTE.—The following report is printed here prior to its being submitted to the full Committee in order that criticisms and suggestions may be received in time for the Committee to be able to consider them along with the Report.

All comments should be addressed, as early as possible, to the Hon. Sec. of the Sub-Committee, F. S. Macaulay, 19 Dewhurst Road, Brook Green, London, W.]

The Sub-Committee appointed to consider the course of Mathematics required for Entrance Scholarships at the Universities makes the following recommendations:

1. It is very desirable that the subjects of examination for Mathematical Entrance Scholarships at Cambridge and Oxford should admit of the same course of preparation.

2. In order to allow of a broader syllabus the exercises and riders should be much simplified.

3. Questions on elementary bookwork ought to be set. Restricting bookwork questions to the higher bookwork would encourage the neglect of the more elementary parts, which are of at least equal importance.

The Sub-Committee considers that the essay type of bookwork questions (as set in the Trinity Group of Colleges at Cambridge in December, 1905) is of special value.

4 A knowledge of the general principles and application of graphical work should be required, but only a moderate degree of manipulative dexterity.

5. The range in Pure Geometry should approximate to the Oxford standard: purely geometrical methods should be encouraged by a fair number of questions being set which can best be solved by pure Geometry. Purely geometrical methods as opposed to analytical should not be insisted upon, but there is no objection to the appropriate method of solution being recommended.

6. The number of questions set in Algebra and Trigonometry should be proportioned roughly to the time required for the study of the two subjects.

The Sub-Committee considers that too little credit is given for Algebra in some Scholarship Examinations.

7. In the Calculus, geometrical proofs, which assume that the functions considered are such as can be represented by curves, should be permitted, provided that the necessary conditions for their validity are stated.

8. In Geometry, Trigonometry, Theory of Equations, and Dynamics the use of Infinitesimals and the methods of the Calculus should be freely permitted.

9. The subjects of the examination should be in accordance with the Schedule appended, due regard being paid to the second recommendation of the Sub-Committee.

SCHEDULE.

PURE GEOMETRY.

Geometry of Straight Lines, Circles and Conics; Inversion, Cross-ratios, Involution, Homographic Ranges, Projection, Reciprocation and Principle of Duality; Elementary Solid Geometry, including Plans and Elevations.

ANALYTICAL GEOMETRY.

Straight Lines and Curves of the Second Degree; Homogeneous Coordinates; Tangential Coordinates.

Excluding the application of homogeneous coordinates to difficult metrical questions; Invariants; and Analytical Solid Geometry.

ALGEBRA, including Elementary Theory of Equations.

GEOMETRICAL TRIGONOMETRY. *Excluding* Spherical Trigonometry.

ANALYTICAL TRIGONOMETRY.

Properties of circular, hyperbolic, exponential and logarithmic functions, with real and complex argument.

Excluding the *proofs* of the Infinite Products for Sine and Cosine, and of the Series of Partial Fractions for the other Trigonometrical Ratios.

CALCULUS.

Total and Partial Differentiation; Taylor's and Maclaurin's Theorems; Elementary Integral Calculus; Simple Applications to Plane Curves (especially to such as are of intrinsic importance), to Maxima and Minima, to Areas and Volumes, and to Dynamics; Curve tracing, not as a rule to scale.

Excluding Differential Equations.

DYNAMICS.

Elementary Statics, including Simple Graphical Statics; Elementary Kinematics and Kinetics, including motion of a rigid body about a fixed axis, and motion of cylinders and spheres in cases where the centre of gravity describes a straight line.

Excluding Hydrostatics and Hydrodynamics.

THE FUTURE OF THE MATHEMATICAL ASSOCIATION.

SINCE my article on "The Neglected Teacher" appeared in the *Gazette*, I have received a number of letters favourable to the proposals there sketched out.

Prof. Alfred Lodge, M.A., Charterhouse, Godalming, writes:

I have read your article in the *Mathematical Gazette* with immense pleasure. It opens up vistas of usefulness, *e.g.* it is to be an intelligence bureau for answering questions. Now two or three questions I want to ask are:

(1) What is best book for studying convergency of series (with sufficient illustrative series) for school purposes?

(2) What is best book for trilinear and areal coordinates?

(3) What is best book for tangential coordinates?

At Coopers Hill we did nothing of this sort, so one is naturally rusty. My colleague, Tuckey, is good on them all, but I would like to be happier on them for my own sake. Can you give me your ideas?

Also (4) where can one get hold of first notions on theory of groups, and (5) on theory of functions? To No. (4) I expect you will say Hilton. If so, I will certainly get it.

We have some very good mathematical boys here, and after they have secured their scholarships there are still two terms in which they are with us before going to college. A good mathematical library of not too difficult books is just what we want.

Again, as regards the *Intelligence Bureau*, if half a dozen sub-editors each engaged to examine new books with a view to culling *new ideas, suitable for school purposes*, in connection with definite mathematical regions,—*e.g.*

Pure Algebra, such as series, continued fractions, etc.
Higher Trigonometry.
Calculus, including reference to such pretty problems as your one in *Nature*.
Statics and Dynamics. (There is a new book by Jackson & Milne on Statics—for beginners—which is a vast improvement on the usual run.)
Analytical Geometry.
Pure Geometry.

—and if they published the results of their observations in the *Gazette* it would much assist men who are preparing scholarship candidates.

Perhaps some similar work might be done for elementary mathematics for general school work—in fact, Jackson & Milne's book belongs rather to this category. But in this connection the best assistance, to my mind, would be given by sets of simple illustrations of use of mathematics *drawn from real life*, so that by degrees the text-book examples would become more living and concrete. At present, what is called Practical Mathematics, is too fragmentary, too much divorced from the regular routine of upward progress to be much use for schools.

We suffer in schools from lack of time and lack of money to attend meetings of learned societies. It is a pity, because quite good men come to us red hot from their university training, and cannot keep it up by the natural method of rubbing against their peers or their leaders personally. A résumé of progress in some branch or another of mathematical investigation, written by a leader *in sympathy with* those who are thus out of personal touch, but are still keen students, would be a vast help and inspiration. If we succeed in increasing our membership as suggested by you, the *Gazette* could be enlarged, and much of what I have outlined might be done.

It would encourage us to form little mathematical associations of our own, each at his own school, or at groups of neighbouring schools, for mutual discussions and reading of papers, by giving us the needful ideas to work upon.

Mr. C. H. Blomfeld, M.A., the Grammar School, Bradford, writes:

I hope the suggestions you make with regard to the conduct of the *Gazette* will be carried out. I have been a member of the Association for several years, and with the exception of the reports and discussions of the reports of the committee for the reform of the teaching of Elementary Mathematics, I have rarely found anything of general interest. The whole *Gazette*, it appears to me, is taken up with special solutions of problems of too advanced a type for the ordinary mathematical work of a school. These ought, I think, to be there in due proportion, but not to the exclusion of the rest. For instance, at the present time the great problem in a school like this is to get the Geometrical teaching thoroughly done—not the advanced teaching (I mean Casey, etc.), but the ordinary Euc. I.–VI. The chief result of the recent changes is that an additional burden has been flung on our shoulders; namely, teaching geometrical drawing as well as propositions and riders, all to be crowded in two periods of from 40 to 45 minutes a week at best. Now during the whole of the time since the New Geometry has been started there

has been (I believe I am correct in saying) absolutely no assistance from the *Gazette*, which is still mainly concerned with the kind of problem and proof useful for boys who have obtained, or are just about to obtain, scholarships. I have just been looking through the back numbers which I possess, and the impression given is a somewhat wearisome recurrence of more or less fantastic series. I see there is an article on the teaching of Geometry in the current number, but what is the use to anyone who has not been accustomed to use them of the following: "Models of cardboard and string, wire models and hatpins stuck through cardboard have their special uses" (what models, and how made, what uses ?). Examples of this might, I think, be multiplied indefinitely.

Mr. C. St. John Shortt, M.A., Sandroyd School, Cobham, writes:

I have been more interested in this March number of the *Gazette* than before. I must confess that most of the contents of previous numbers has been quite beyond me. I have no doubt that it appeals to, perhaps, the majority of the members of the Mathematical Association. But I imagine there is a sprinkling of members who, like myself, are engaged in Preparatory School work, who have little time and perhaps little understanding for learned dissertations on say "Univocal Curves" or "Illegitimate Differentiation." On the other hand, a valuable note like that of Lodge's on "Contracted Methods" in the January number was of great assistance to me, and I really should welcome simpler material of this kind. I think the idea of founding local branches of the Association is an excellent one.

These letters and Mr. Child's paper are exactly the kind of discussions which ought to be encouraged in our *Gazette*. I have never been sanguine that the so-called "reform of mathematical teaching" would leave us much better off than we were before, and the present is an opportune time for discussing such questions. Personally I consider that reform of teaching means something more than mere tinkering with syllabuses, the publishing of school geometries, and the writing off of publishers' losses on the innumerable Euclids that came out just before the change. The teaching of algebra is at present in a hopeless chaos, and the subject has practically all but disappeared from our curricula.

The old kind of useless drudgery which was called algebra is quite unsuited to modern requirements. It disgusted me so with algebra when I was a boy that I lost heavily in every algebra paper in the college and university examinations in my student days. There are many useful things which could be taught to boys under the heading of algebra, but it is impossible to teach things which are not examined on or to examine on things which are not taught, or to teach and examine on things without some text-book as guidance. Here, then, we have another subject on which the opinions of teachers would be valuable, and in order that the discussions may be effectively carried out we must induce the rank and file of the teachers to join the Association.

A few years ago one or two discussions on teaching mathematics were organised in Bangor by Professor Green, and what struck me most about the opinions expressed by those present was the deadly effect of syllabuses in stifling all efforts at originality on the part of the teachers. Unless mathematics is to be taught as a living reality which enters into every problem in the life of the nation, then, I say, we might just as well abolish school mathematics altogether and substitute philately in its place.

I consider that the recent papers on convergent series in the *Gazette* fulfil one very useful purpose. They show the futility of attempting to teach things like the Binomial Theorem to the ordinary schoolboy. But if anyone complains of the somewhat advanced character of the contents of the *Gazette* the Editor will tell him that he is only too glad to get papers and discussions of a more elementary character, but that people will not write them.

There must be a great many teachers who, however hard they are worked, could write *some little thing* in the course of the year to keep the ball rolling now it has been started. It is impossible for one man to keep a ball rolling indefinitely, or even to start it rolling at all when there is a large amount of inertia to overcome, but a little united effort would do much in rescuing the study of mathematics from the despised position which it now occupies in the eyes of the British public. G. H. BRYAN.

A FIRST PRACTICAL TRAINING IN ARITHMETIC.

THE following practical suggestions are offered as an answer to the question of ever-increasing interest, "How, when, where, shall the child begin arithmetic?"

The importance of the science of number, as a great teacher has shown, is enormous, not only in its relation to the study of abstract mathematics and in its dominance over the prosperity of nations, but in the fact that it is the "conclusive science which we have to apply all our days to all our affairs."

Herein lies the clue towards a right method of presenting it to children. The training which we owe them in this direction must be an eminently practical one, based on active interest in every-day life and surroundings. It will begin as soon as the child's activities need an outlet, and he shows that he is ready for intelligent play.

He will spend two or three years during the earliest stages developing ideas of number up to twenty, by means of varied practice in arranging objects, such as bricks, dominoes, cards. He will gradually become familiar with the coinage by means of a box of "play" money, which he will use to buy and sell in the nursery market. Notions of weight will be learnt from the use of scales, as ideas of length are gained by much practice with foot and yard measures and rules. He will be taught simple games that involve recognition of the number of pips, such as "Grab," "Old Maid," and the easiest form of dominoes. When he shows himself ready, he can be led to transfer his ideas of number to figures.

In the two next stages the same interest is kept up in games, in buying and selling with the coins. He is also introduced to dry and liquid measures in a practical way; his memory and judgment will be exercised in every possible manner as opportunity offers. His limit of number is still twenty; and when he is quite ready he works, entirely with his teacher on the blackboard, every addition, multiplication, subtraction, and division sum (up to twenty), using the necessary arithmetical signs. The use of objects is dispensed with when no longer needed. He next passes on to do the same sums on paper, entirely by himself.

This brings the child to the fifth and sixth stages of his course, when his number knowledge will be increased (by concrete illustration) first to thirty, then to sixty, so that he can extend the working of the sums he already knows, and also the writing out of tables, to these numbers. Then, working from very easy bills, he will learn how to set compound and simple addition and multiplication sums. The process of "carrying" will offer little difficulty after his experience in changing coins in the play-shop keeping, and in the constant use of scales and measures. At this stage a clock face with movable hands will give much useful practice, and teach besides how to tell the time. This will open up interest in time measure, and in diagrams and models to illustrate the daily rotation of the earth and monthly journey of the moon. Opportunities will be seized to welcome practical help from the child, as he is able to give it, in weighing and stamping for the post, in jotting down small outlays, in measuring or counting. Passing on from the relation of certain fractions, *e.g.* $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, to a foot, a yard, etc., diagrams and models will be freely made and used to give clear ideas of fractions of unity.

In the seventh and eighth stages, realisation of number is brought up to two hundred, chiefly by ruling sheets in a hundred squares, which leads on to square measure. Square feet, yards, and inches are ruled on paper and cut out, and then used to measure suitable surfaces, giving scope for discussion as to carpeting, papering, painting surfaces that can be actually measured. Plan-making naturally follows on, and the necessity and use of a "scale."

The writing out of tables and practice in sums already learnt is extended to higher numbers as before, and the child also learns how to set and work compound and simple subtraction and division. Such practical matters as finding the difference in cost in arranging for some projected outlay interesting to the child, or finding how long a discussed railway journey will take (from the *A B C Guide*), or sharing out savings for Christmas presents, will introduce the sums, and little difficulty will be found in "changing" in subtraction, owing to previous practice with coins, weights, and measures.

The next two stages lead the child to work with numbers up to thousands. By means of an ordinary hoop, with wire or thread tightly fastened across as needed, the meaning of the various measurements of the circle will be shewn, and much practice will follow with compass and ruler. Wire circles to fit the geography globe will give correct ideas of the measurements by degrees on the earth's surface, and great interest can be found in working out questions of distance (in degrees) and in difference of time. To help in the widening of ideas, every opportunity will be taken to point out distant views, from heights, with glasses; and measurements, when possible, will be pegged out on sands or common, such as those of a square pole, or buildings of known size. An acre field, or square mile park, will give bases for estimating areas as need arises. Games have been gradually developed from mere matching to the cultivation of forethought in play, and to their number have been added, by degrees, back-gammon, draughts, cribbage, and later on will follow the elements of whist and the more difficult forms of dominoes. Quickness in recognising factors, prime numbers, measures, will be fostered by daily writing out from the Sieve of Eratosthenes. During these ninth and tenth stages, compound and simple long division, and division by factors, with correct remainders, are added to the knowledge of sums. Practice in cancelling will also be found useful.

In the last two stages of the course thus outlined, realisation of number is brought up to hundreds of millions. Cubic measure is demonstrated by models and diagrams, and much practical work is done in finding the prices of daily needs from store-lists and observation, leading to discussion as to the meaning of such words as "dear," "cheap," "cash," "credit," etc.; also, how best to spend allowances; how to keep accounts. The child will be led to take an interest in commercial maps, year-books, railway guides, by the suggestion of questions in statistics that he can answer with their help, fitting everything in as far as possible with geography lessons. He will also be thoroughly exercised in realising the flight of time, from the forty-fifth century B.C. to the twentieth century A.D., by means of a note-book, each page of which represents a century, and in which he is accustomed to draw or write jottings after history lessons, or visits to museums. For sums in these eleventh and twelfth stages he works on, with higher numbers, at those he already knows, adding the division of one concrete number by another; practice in solving simple problems by the method of unity; answering questions in vulgar fractions by diagrams; and work in elementary decimals.

A training such as this, faithfully and systematically carried through six or seven years, should prove a good preparation for future work; for besides the incidental formation of character, the strengthening of mental powers, the growing insight into the value of time and money, the child has learned

to take real pleasure in work, for he has found out for himself the first answers to the questions perpetually in his mind, "How many?" "How much?" "How far?" "How long?" and he is on the right road to find out more.

FRANCES EPPS.

THE A, B, C OF THE HIGHER ANALYSIS.

1. When Algebra, fair Daughter of King Arithmetic, is nearing her age of adolescence her eyes fall on Prince Limit, and there cometh a change upon her. Her tender heart goes forth to him, and in her lithe and nimble frame she feels a torture from end to end as it would fall to pieces. She is in love, and the Prince reciprocates her love.

2. Happily they are wedded. The King gave away his Daughter, the Angels witnessed it, and all was joy. The marriage was performed with the Hymn of the Transcendent Number and Ratio, preceded by the Song of Limits, and followed by the Chorus of Continuity and Convergence.

3. Algebra is happy for evermore, and with an Amplitude of vision she never knew before, and with her lord by her side, she surveys the vast Domains of Higher Analysis wherein the King's Ancient Laws prevail.

4. Three Children are born of the happy union—all at one birth—and the glad news doth spread in Heaven. By the King's command Newton stood Sponsor and Leibnitz, of felicitous Power Symbolic, conducted the Baptism. The ceremony was grand, and the World witnessed it.

5. But soon as the ceremony was over the silly world started asking, which was the greater on the occasion, the Minister or the Sponsor? And they answered differently as their eyes were priestly or lay, and there was strife. Forgotten be this strife, for Behold, the Children have Eternal Life.

6. Each of the Princes hath features derived from both his Royal Parents, and each also reveals the Imperial form of the King, yet each hath his own characteristics.

7. The first Prince is by habit an Astronomer. He delights in Infinitude. The depths of space and the mysteries thereof engage his attention. The Cometic Systems obey him, and have yielded him the Secret of their Tails. The King creates him Lord of the Infinite or Astronomical Branch of the Higher Analysis.

8. The second Prince is a Biologist, much after his Mother. He delights in the Evolution of Forms. He sees the Life beneath every Form, and thus the Secret of Form he knows, and of that which changeth not with the Change of Form. The King creates him Lord of the Finite or Biological Branch of the Higher Analysis.

9. The third Prince is ever a Chemist. The Atomic Structure of Things delights him and he sees Worlds within Worlds in an atom. He sees the atoms integrate, and in Elemental Atomic Relations he discovers Forms, and Form-Aggregates, and complexities thereof defying Thought. The King creates him Lord of the Infinitesimal Branch, or the Chemistry, of the Higher Analysis.

10. Whoever says that he knoweth his A, B, C thoroughly, Let him stand forward.

11. The little Princes love each other dearly as they do their Parents and their Grandfather, and have grown in each other's love, sometimes playing each other tricks. And now they have gone forth together to conquer the Higher Realms of Functions. Verily they shall have Eternal Life and Eternal Youth.

12. There be some who would set the Princes by the ears, trying to tickle one, and suggesting names to call another by. But the Princes shall not hear them, for Newton is their Guardian.

Gooty, 7th March, 1907. V. RAMASWAMI AIYAR.

THE NEED OF A SEQUENCE IN GEOMETRY.

MESSRS. GODFREY AND SIDDONS in the preface to their Geometry express both a hope and an expectation that a recognized sequence of propositions will ultimately be adopted not widely differing from that of Euclid. In the *Athenæum* of January 9th, 1904, the writer of a review on Messrs. Godfrey and Siddons' book scouts the idea.

Since that time we have seen authority after authority in not only England, Wales, Scotland, and Ireland, but also in Australia and India, gradually falling into line and adopting the syllabus of the University of Cambridge; there also seems to be a tendency towards adopting the *order* of the Cambridge sequence.

I have, therefore, thought the present time opportune to bring forward a point which authors of text-books founded on the Cambridge syllabus *feel* or *notice* most, though it affects to a much greater extent those responsible for the selection of text-books for different examinations, especially in places where two or three different examinations are taken in the school course. The point I bring forward is the proper place, in a sequence, of Euclid I. 47, 48, Euclid III. 35, 36, 37, and, though of course of less importance, that of Euclid I. 35-45.

Many teachers, who still swear by "references" in writing out theorems, bewail the loss of the universally known—if not known, it was the same in all text-books, which came to the same thing in the end—numbered sequence of Euclid; and they would be delighted to see an established definite order of theorems put forward by the universities in unanimity, and generally accepted by everyone. This might be a good thing and it might not. Personally I am inclined to believe it would destroy much of the elasticity gained by the abolition of Euclid; and it certainly would stereotype in text-books and regulations those errors of Euclid's "Modern Rivals" (!), which experience alone will eradicate, the like of which caused Euclid to be generally abandoned. One of these points was the incongruous grouping of some of his propositions, due in nearly every case to expediency, owing largely to the long chains of connected theorems and problems dependent on them. The two worst cases of this incongruity are still retained, owing to examination syllabuses.

Thus, in the syllabus for Mathematics, Stage I., of the English Board of Education, the ground to be covered in geometry is set as "equivalent to Euclid, Bk. I.," whilst the Oxford and Cambridge Preliminary Local Examinations set their elementary or Part I. papers on "equivalent to Euclid, Book I., Propositions 4-6, 8, 13-16, 18, 19, 26-30, 32-41, 43," thus carefully excluding I. 47, 48. Again, the syllabus for the English Board of Education, Stage II., requires Euclid II., III., IV., as does the syllabus for London matriculation and other examinations of this type. The syllabus for the Scotch Leaving Certificate is "the equivalent of Euclid, Books I. and III." only, including, however, Euclid III. 35, 36, 37 (though not Euclid II.!), whereas the Oxford and Cambridge authorities ask for Euclid I. and III. and IV. 1-5, *without* III. 35, 36, 37, as also does the College of Preceptors, and even the Board of Education in King's Scholarship papers.

Many other examples could be found by examining the syllabuses of the more modern universities of Great Britain and the colonies, though chiefly centred on the placing of Euclid I. 47, 48, III. 35, 37. Some definite arrangement should be made to bring all examinations of the same type and class into accord, and do away with the chaos that now exists. Whether this will take the form of a definite numbered sequence in the near future remains to be seen. If this were to happen, the ideal arrangement, in my opinion, would be totally different to any which now exists; and in this belief I feel I should have with me all practical people, the "base

utilitarians" who teach geometry for geometry's sake alone, and also many of the theoretical mathematicians who take geometry as the epitome of logical training. I mean that, since it is recognized that the strict treatment of ratios and proportion is not suitable for elementary text-books for beginners, Euclid, Book VI. 1-12, ultimately depend on the proposition about two transversals falling across three equidistant parallels, now included in the equivalent to Euclid I. Hence as soon as Euclid I. 4, 8, 26 have been proved, the corresponding theorems on similar triangles should be introduced, and so complete the batch of propositions on triangles with three parts given. The difficulty of introducing such a change is non-existent; ask any teacher of geometrical drawing whether he ever had any difficulty in getting a beginner, who had enough νοῦς to understand the bisection of a line, to understand the construction for dividing a line into any number of parts. Why, even now we have in elementary practical work the division of a line in a given ratio, mean proportional, scale construction, etc. See what an easy proof one gets for Euclid III. 35-37: almost self-evident theorems when proved by equiangular triangles. There is no theoretical difficulty, and what a practical gain!

That this arrangement should meet with universal acceptance by all examining authorities is, however, beyond the bounds of probability, I had almost said possibility. Unless this arrangement were the one universally accepted it would work more harm than good to have a definite numbered order, which is subject to the contention that, no matter what its theoretical interest, it is not the best practical order.

However, one thing could easily be done by general agreement between the leading examining bodies—the minors would perforce fall into line or get left under stress of school curricula--and that is a settlement of the logical position of certain propositions and a definite division of the whole scope of school geometry.

Euclid I. 47, 48 go naturally with Euclid II. 12, 13—in fact, the latter are easily proved by an exactly similar figure to that of Euclid I. 47, a method of proof which has many points in its favour. Again, Euclid III. 35, 36, 37 follow naturally on II. 12, 13, and should go with them; whereas the elementary congruences and tangent properties of the circle follow naturally on those of the straight line, and should come, in my opinion, before the area propositions I. 35-43.

Thus, although a definite *numbered order* may be undesirable, yet a definite *grouping* is highly desirable. Surely those who quote the Cambridge syllabus could not do better than follow the Cambridge lead, slightly amended, and accept the groups:

I.	Straight line,	Euc. I. 1-34.
II.	Circle,	Euc. III. 1-34.
III. A	Areas,	Euc. I. 35-43.
III. B	Areas,	Euc. I. 47, 48, II., III. 35, 36, 37.
IV.	Similar figures,	Euc. VI.

It would mean a very small change in a comparatively few syllabuses, and would be to the advantage of everybody. The only people it would affect are the authors of text-books designed for certain examinations, and these text-books could easily be changed; and speaking as one of the authors of such text-books, I think we should welcome the necessity of having to make the change. J. M. CHILD, B.A., B.Sc.

MATHEMATICAL NOTES.

237. [V. 1. a.] *Contracted Multiplication and Division.*

Mr. Godfrey and Prof. Lodge have both expressed the hope that others would give the results of their experience of contracted methods of multi-

plication and division. My experience agrees with Mr. Godfrey's. That the rules are to nearly all boys pure "rule of thumb" is shown by the number of different dodges which the upholders of the methods advocate, each maintaining that his own particular dodge is *the* one which renders the rule easy to remember and safe to use. One or other of these dodges is learnt with much pain and forgotten again in a few weeks. In some cases it is desirable to teach "rule of thumb." I suppose we all learnt multiplication and division in that way, but here the value of the result was such as to justify the means. It is necessary to know how to obtain a product or a quotient at an early stage of our career; but it is not necessary to know how to obtain a product to a given degree of accuracy with the least possible amount of writing. A boy who really understood what he was doing could reinvent a method for himself if the possibility were suggested to him; he probably would not invent the shortest, but is it desirable to spend all the time we do merely to ensure that such a boy shall obtain his result by writing down 36 digits instead of 40?

As for the practical utility of the method, I do not believe that any computer would habitually use it. If he had a number of multiplications to perform he would use a slide rule, or logarithms, or some such tables as Crelle's, or a calculating machine, according to the nature of the work on which he was engaged. It is only for an occasional multiplication that turns up when none of these aids are at hand that the method would be employed, and, unless he were far more familiar with it than is the ordinary boy, the expenditure of brain power involved would more than neutralise any slight gain that might otherwise accrue. When the method is insisted on in school we have probably all met the case of the boy who writes out the work in full on some paper he hopes will not be discovered, and then copies out just that part of it which he thinks his instructor will like to see. One of the aims we profess to have set before ourselves in our recent reforms has been to teach boys what is interesting and what is likely to be practically useful. The practical utility of these methods—if any—ceases for a boy when he begins to work with a slide rule or logarithms. I have yet to meet the boy who will even pretend that they present any interest whatever to him.

I am sure that the time and labour devoted to these contracted methods might be much more advantageously employed. We have to frame our course to suit the average, or even the stupid boy, and I fear that in any case we shall leave him "unable to deal neatly with masses of figures out of which he requires to obtain a result to a moderate degree of accuracy" when he is denied access to a logarithm book. Let us rather try to teach him something which experience shows he has a chance of understanding, or of remembering, and which is more likely to be of some practical utility.

With regard to the way of writing out a logarithmic evaluation, I should like to suggest a slight alteration in Prof. Lodge's method.

To take his example: $x=\dfrac{3{\cdot}142\times 73{\cdot}28\times 5{\cdot}923}{48{\cdot}34\times 39{\cdot}67}$.

I should teach a boy to work thus:

$$\begin{array}{rll}
\log 3{\cdot}142 = & {\cdot}4972 & \\
\log 73{\cdot}28 = & 1{\cdot}8650 & \\
\log 5{\cdot}923 = & \underline{{\cdot}7725} & \\
\log \text{Num.} = & & 3{\cdot}1347 \\
\log 48{\cdot}34 = & 1{\cdot}6843 & \\
\log 39{\cdot}67 = & \underline{1{\cdot}5985} & \\
\log \text{Den.} = & & \underline{3{\cdot}2828} \\
\log x = & & \bar{1}{\cdot}8519 \\
x = & {\cdot}7110. &
\end{array}$$

The use of two (or sometimes three) columns obviates the necessity for copying the logarithm of the denominator, and copying is the most fruitful source of mistakes with which I am acquainted. S. A. SAUNDER.

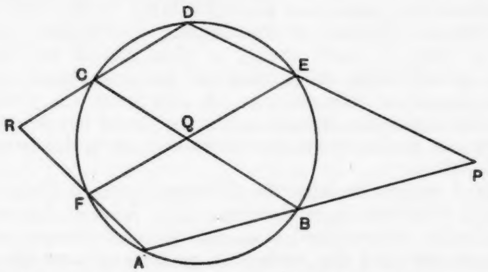
238. [K. 9.] *Geometrical Illustration of Pascal's Theorem.*

Let $ABCDEF$ be a hexagon inscribed in a circle whose opposite sides (AB, DE), (BC, EF), (CD, FA) intersect in P, Q, R respectively.

Then perps. from P, A on QB are as $PB : AB$.

But diameter . (perp. from A on QB) $= AB . AC$. [Euclid VI. c.]

Therefore diameter . (perp. from P on QB) $= PB . AC$.



Similarly diameter . (perp. from P on QE) $= PE . DF$.

Therefore perps. from P on QB, QE are as $PB . AC : PE . DF$, or as $BD . AC : EA . DF$; since triangles PBD, PEA are similar.

In the same way perps. from R on QC, QF are as $CA . DB : FD . AE$.

Hence P, Q, R are collinear. R. F. DAVIS.

REVIEWS.

Vorlesungen über mathematische Statistik (Die Lehre von den Statistischen Masszahlen). By E. BLASCHKE. pp. viii, 268. 1906. (Teubner.)

Dr. Ernst Blaschke, who is professor at the Royal Technical High School in Vienna, has produced, in the volume under review, a valuable and comprehensive synopsis of the mathematical methods applied to the modern scientific development of statistics. Primarily the book is intended for students, but we are disposed to think its worth will be even more appreciated by experts, who will here find, condensed into the relatively brief compass of 268 pages, a description of some of the methods most recently adopted, as well as of others employed in the past, when even experts were seeking to establish first principles. In one sense the book suffers somewhat from its very compactness, since important mathematical formulae are introduced and discussed without much preliminary demonstration. We are not able to offer any opinion as to the attainments of Austrian mathematical students, nor as to the extent in which Dr. Blaschke's apparent faith in their preliminary training is well founded, but in his preface he expresses the opinion that not only the insurance expert, but also the managers of large industrial undertakings, as well as lawyers, judges, and doctors, are closely concerned in insurance matters, and that the study of insurance

principles is a valuable part of their education. That may be the case, but the suggestion savours somewhat of a counsel of perfection. Probably many highly successful insurance managers have not been experts in the higher mathematics, and it is almost too much to expect members of other professions to possess the standard of mathematical knowledge requisite for a due appreciation of a text-book of the kind.

The work is divided into six sections. The first section deals with statistical groups or aggregations of individuals among whom the same characteristics or variations are observable, and the measurement of those characteristics, numerous examples being drawn from mortality tables. Various methods of representing the number of persons exposed to risk of death during a given period are introduced, including several based on applications of geometrical figures not usually employed in this country. A comparison is given of the methods of obtaining the exposed to risk employed in the construction of German and English mortality tables and the tables issued by the Gotha Life Office.

The second section explains the difference between "intensive" and "extensive" statistical measurements. The function known as the force of mortality is mentioned as an example of the former, and among the latter are included the arithmetic, geometric, harmonic and anti-harmonic means, the central value, and the mode. As special instances are quoted, of the arithmetic mean—the average duration of life, of the central value—the probable duration of life, and of the mode—the normal age. A discussion follows of the method of obtaining the exposed to risk from the records of insurance offices, and a description of the material used in the construction of the principal English and foreign mortality tables, annuity tables, invalidity tables, sickness tables, and tables of probability of marriage, with comparisons of percentages derived therefrom.

In the third section the author discusses and compares statistical and mathematical probabilities, introducing theorems of Bernoulli, and Gauss's formula for the law of error, and dealing with normal dispersion and the law of variation of extensive measurement.

Section 4 traces the development of the deduction of statistical laws, starting from the hypotheses of De Moivre and others respecting the law of mortality, and proceeding to the well-known formula of Gompertz and its development by Makeham. The law of error as demonstrated by Pearson and by Gauss is also discussed.

The fifth section is devoted to the application of the doctrine of probability to insurance and the employment of statistical results. It includes a discussion of the insurance premium, mathematical risk, and the applications of its theory.

In the sixth section methods of graduation are defined and described, special mention being made of King's method of applying the Gompertz-Makeham formula and of Pearson's method of moments. The section concludes with a discussion of mechanical methods of graduation, including those based on finite differences and on graphic representations.

The first appendix contains an illustrated description of machinery

designed for counting and perforating cards used in statistical investigations, and the second consists of a table of the value of the function

$$W\gamma = \frac{2}{\sqrt{\pi}} \int_0^r e^{-t^2} dt.$$

The book closes with a bibliography of statistical literature.

The volume is a noteworthy contribution to statistical science, meriting careful study, and would probably meet with wide appreciation if translated into English and perhaps elaborated by a fuller demonstration of the mathematical formulae employed. W. R. STRONG.

A College Algebra. By HENRY BURCHARD FINE. viii + 595 pp. Boston: Ginn & Co. [1905].

After (pp. 1-5) introducing the finite cardinal numbers much in the way of G. Cantor, Prof. Fine begins a theory of what he calls (p. 6) "natural numbers," which are "signs for the cardinal numbers," and introduces negative, fractionary, irrational (by a theory most allied to Dedekind's), and complex numbers as new marks of order interpolated in the ordered system of these "natural numbers" (pp. 17, 32, 59, 70). This empty nominalism of a theory of "signs" is at bottom the same as that of Helmholtz, Kronecker, and many other amateurish philosophers among mathematicians; its weak points have been admirably ridiculed by Frege,* and Cantor † has shown that it constitutes, by its beginning with the most unessential part of the theory of number, a ὕστερον πρότερον. It may be remarked that Peano never stated that the *sign* 1 is a number, as he is said to have done on p. 12. Of course, he recognised that the sign 1 is a sign, but not a sign for a sign.

Thus, pp. 6-78, with their careful proofs of the commutative, associative, and distributive laws in all the cases, seem to me spoilt at the outset. Formalism in mathematics is difficult to get rid of, and it is a serious matter for students that such theories should be countenanced, as they are by many writers of text-books.

The rest of the book is devoted to algebra in a wide sense—equations, rational, irrational, and symmetrical functions, binomial theorem, inequalities, indeterminate equations of the first degree, progressions, interpolations, permutations and combinations, probabilities, infinite series (including some theorems on power-series) and products, and some theorems on continuous functions (including a proof of the fundamental theorem of algebra). This part is often happily original, and on the whole thorough, though some parts (such as the section on infinite products) are hardly so much so as is desirable.

PHILIP E. B. JOURDAIN.

A First Course in Statics. By C. S. JACKSON, M.A., and R. M. MILNE, M.A. London: J. M. Dent & Co. 1907. pp. 380, with 200 woodcuts. Price 4s. 6d.

In the discussion which took place on the Teaching of Mechanics at Johannesburg in 1905, under the leadership of Professor Perry, the

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* *Die Grundlagen der Arithmetik*, Breslau, 1884, p. 107.
† *Zur Lehre vom Transfiniten*, Halle, 1890, pp. 16-19.

one point which stood out most conspicuously from a collection of opinions, many of them diametrically opposed to each other, was the generally expressed view that the teaching of mechanics should be more experimental and practical, less dogmatic. If any other points were noticeable, one of them was that there was not the same divergence of opinion about statics as about dynamics. The present book appears admirably adapted to its object of making statics a living reality, and not a mere edifice of human imagination. The necessity for such a change is amply proved by the fact that the present reviewer has *actually heard* Cambridge graduates with high mathematical degrees admit that a difference exists between results which are mathematically true and results that are in accordance with experience, and a question set some years ago on snow-sliding down a roof afforded an instance in point.

As examples of the many interesting and useful features of the book, we note the following: Reproduction of Egyptian bas-relief showing the balance (p. 5); early experimental discussion of the principle of the lever (p. 7); wheelbarrow and wire nippers (p. 25); early introduction of resistances (p. 44); Stevinus' early discussion of inclined plane by supposing a necklace hung round it (p. 62); illustration of moments from crank and connecting rod (p. 94); extended discussion of velocity ratio and efficiency (chap. v.); experimental study of friction (chap. vi.); introduction of Newton's First Law in discussing friction, a cause which makes that law *appear* incorrect; good figures in illustration of Guldinus and other mensuration problems; force diagrams, including Warren girder and roofs (pp. 244 to 326); use of "Plan and Elevation" diagrams in dealing with forces in space (pp. 349, 351); example on grab used for lifting stones, taking account of friction (p. 372).

These features are probably as many as could be judiciously introduced at the present time. Perhaps one of these days the authors will consider the following suggestions for a future edition: To put resolving before Lami's Theorem, thus enabling statical and geometrical proofs to be discussed side by side; to bring the principle of work still earlier to the front, possibly at the very beginning in connection with the lever; to tell us more about friction, in particular, differences between rolling and sliding friction; whether locking the wheels stops the train most efficiently; whether friction depends on the grain of the rubbing surfaces; whether the cone of friction is always a right circular cone (elementary experiments to be suggested for class teaching); and a few other matters of the same kind.

At any rate we have got rid at last of the three levers and the three systems of pulleys which have so long occupied a place side by side with the statement that various things "are of two kinds, simple and compound."

The most disappointing feature is that with very few exceptions the examples are expressed in terms of such obsolete units as lbs., cwts., tons, and feet, which have long disappeared from all civilised nations in favour of the metric system. The old Cambridge problem, in which the answer "may be put in the form" $P = 2W \cos\alpha \operatorname{cosec}(\theta - \beta)$ "where" $\cos\beta = \cos\alpha\cos\theta$ was at least free from this objection. In view of the

existing chaos regarding units, Messrs. Jackson and Milne have been wise in confining their attention to statics, and not extending the sphere of their operations to dynamics. G. H. B.

Lehrbuch der Mechanik. 1 Teil. **Kinematik.** By Dr. Karl Heun, Professor an der Technische Hochschule in Karlsruhe. (Leipzig: G. J. Goschen.) 1906. pp. xvi, 339. Price 8 marks.

The complete work, of which this is the first part, is intended as a presentation of higher dynamics, and as a book of reference for technical students.

A quotation indicates the spirit in which Prof. Heun has approached his task: "As mathematical aids to the investigation of a problem in mechanics, the methods of geometry and analysis in all their vast extent are at our service. But which tools from the immense storehouse shall we grasp as being the most useful? A man whose mathematical knowledge is so limited that the choice of means presents no difficulty, is not in reality in an unfavourable position, but he is most to be envied who has always considered the value of the simplest mathematical tools, and has trained himself by the continual use of these few tools" (p. 4).

The elements of the calculus and vector algebra are the tools used here. The treatment is vectorial throughout, not merely in the sense of M. Appell's great treatise, that is, physically, but mathematically. As to notation, a vector is distinguished by a bar, $\bar{a}$; the scalar product, termed work product (arbeits produkt) is written $\bar{a}\,\bar{\beta}$; and the vector product (moment produkt) as $\overline{a\beta}$. Dr. Heun agrees with Professor Tait in thinking that in many cases a vectorial formula suggests its own physical significance almost as clearly as a model could do. The technical student, however, beyond all others, always ends with arithmetic. That is, whatever analytical processes he may employ on the way, his goal is always some number. Dr. Heun has fully appreciated this fact, and realises that, however concise a vector formula may be, it can never furnish the final answer to a problem, and that when numerical results are wanted the services of Messieurs X, Y, and Z are indispensable.

In the first 112 pages, after some preliminary work in geometry, the kinematics of a particle are dealt with. Next the motion of two connected particles is considered, and the question is put: Are these particles geometrical points distinguished only by difference of position? No, we must distinguish a *material* from a geometrical point, attributing to the former mass a numerical coefficient of the velocity or of the acceleration vector. This mode of treatment avoids several metaphysical topics with which the discussion of mass is usually associated.

In the third section, Euler's equations and moving axes, with special reference to the motion of the earth, and, in general, the kinematics of a rigid body, are considered. The least satisfactory part of this section appears to be that dealing with moments of inertia, where the suffix notation employed gives a needlessly complicated appearance to some of the formulae. To adjust the conflicting claims of the most elementary method, the most obvious method, the most elegant method, and the most universal method of dealing with a problem is a difficult task for an author. Professor Heun has preferred the claims of uniformity of method, and it is not to be doubted that many students will be grateful to him for shewing how much can be done with limited mathematical resources. C. S. Jackson.

The Scientific Papers of J. Willard Gibbs. Two Vols. £2 2s. net. (Longmans.)

Josiah Willard Gibbs was born in New Haven, Connecticut, on February 11, 1839, of a family sprung from the early Puritan colonists of Massachusetts. His father, likewise Josiah Willard Gibbs, was a professor in the Divinity School at Yale, and his ancestry on both sides had for many generations been connected with the two most ancient and most celebrated of American Universities.

After a course of general distinction at Yale, followed by three years of somewhat miscellaneous teaching in the College, Gibbs, in his twenty-seventh year, set out for Europe and spent some time at Heidelberg, then in the zenith

of its fame. Kirchhoff and Helmholtz were professors together; ten years previously, Kirchhoff had discovered the elements in the solar atmosphere by spectrum analysis, and the intervening decade had seen the publication of Helmholtz's *Physiological Optics* and his *Theory of Tone-sensations*. The discoveries of Thomson, Stokes, and Clerk-Maxwell in England were eagerly studied and discussed in Germany; and the young American readily allowed himself to be carried away by the prevailing enthusiasm, and resolved to devote his life to the advancement of Mathematical Physics.

Shortly after his return from Europe, Gibbs was appointed to a full professorship in his old university; and two years later, at the age of thirty-four, he published his first paper, "On Graphical Methods in the Thermodynamics of Fluids." This was followed shortly by a second memoir, "A Method of Geometrical Representation of the Thermodynamic Properties of Substances by Means of Surfaces," and in 1876-8 by his greatest work, "On the Equilibrium of Heterogeneous Substances."

It is scarcely too much to say that these investigations created a new department of science. But recognition came slowly. The *Transactions of the Connecticut Academy* was a somewhat obscure periodical, and no previous researches had given Gibbs a claim on the attention of the world. Moreover, the papers are very long—the third paper occupies over three hundred pages of the reprint—and are difficult reading even to modern students trained in the ideas of physical chemistry. In 1878 the average chemist had little acquaintance with mathematics, and the average mathematician had little interest in chemistry. Thus it was not until the group of memoirs was translated into German in 1891 by Ostwald that scientific men fully realised the epoch-making character of the work.

While the contributions to thermodynamical chemistry must be regarded as the greatest achievement of Gibbs's career, the volumes before us bear ample testimony to his interest in other branches of mathematical physics. The second volume is occupied chiefly with Vector Analysis—including his Theory of Dyadics—and the Electromagnetic Theory of Light. The professorial lectures on the former subject have already been published under the editorship of Dr. E. B. Wilson, and are not here reproduced. The work on *Statistical Mechanics*, which appeared towards the end of Gibbs's life, is also not reprinted.

At the end of the second volume is a biographical sketch of Gibbs's colleague, H. A. Newton, which deserves to be studied as a model of what such a sketch should be. At the beginning of the first volume is another excellent biography, that of Gibbs himself, written by Professor H. A. Bumstead. It gives a pleasant picture of the quiet unassuming scholar, his simple life at New Haven, and the affection with which he was regarded by those who had the privilege of his guidance at the outset of their own work. E. T. W.

Introduction to Infinitesimal Analysis: Functions of one Real Variable. By Oswald Veblen and N. J. Lennes. New York: John Wiley & Sons; London: Chapman & Hall, Ltd. 1907. vii + 227 pages.

"A course dealing with the fundamental theorems of infinitesimal calculus in a rigorous manner . . . appears in the curriculum of nearly every [American] university. . . . This little volume is designed as a convenient reference book for such courses" (p. iii). And the first chapter (on the system of real numbers*) shows the influence on a text-book of those logical considerations on the principles of mathematics which are, of course, essential to mathematics. Thus, on p. 3, we read that "the essential step in passing from ordinary rational numbers to the number corresponding to the symbol $\sqrt{2}$ is thus made to depend upon an assumption of the existence of a number a bearing the unique relation just described to the sequence a_1, a_2, a_n, ..."; and this statement is to be welcomed as almost the first explicit acknowledgment in a text-book that a real number is to be defined at all, though it seems that the authors carefully avoid

* This chapter is not intended to be a full treatment of the theory of real numbers, but chiefly a *classification* (p. 1) of them into rational, algebraic, and transcendental. Proofs are given (pp. 19-29) of the transcendency of e and π.

the (logically advantageous) assumption that this relation is "is identical with" (cf. the note on p. 11, in which the identification of a fraction with an integer-pair is expressly refrained from). Again, an account is given, on pp. 11-15, of definitions of the number-system by groups of axioms. This is to be welcomed on the above grounds, although more modern investigations have proved that a group of axioms does not, in general, determine a concept uniquely, and hence does not define it at all. *

The most novel point in the book is the stress laid upon, and the elegant applications of, what is known as the Heine-Borel theorem. Some time ago the authors † had remarked that this theorem is an equivalent of Dedekind's axiom ‡ of continuity, and can replace the much-longer Bolzano-Weierstrass process of subdivision for the purpose of finding a limit-point; and in this work (cf. pp. 35-40, 48, 88-90, 158) this point of view is developed. It is often true that shortness and elegance in a proof implies that the real nerve of the proof is concealed, and it seems that either the method of subdivision or Dedekind's method is, on this account, to be preferred to the elegant indirectness of applications of the Heine-Borel theorem. Consequently, the additional proof (pp. 41-43) of the theorem on the existence of a limit-point, by subdivisions, is most welcome. Another remark is of more fundamental importance; if there are *many* intervals σ which contain a certain point x, in the Heine-Borel theorem (as admitted on pp. 33. 34), of which no σ has any special property, such as of being the *greatest* of those belonging to x, the selection of a set of σ's containing all the x's may lead § to an assumption of that multiplicative axiom which Zermelo ‖ had the merit of formulating explicitly, which it is a great temptation to most of us to assume, and which may be easily avoided in Weierstrass's process.

The treatment of limits (chap. iv.) has the advantages both of conciseness and of clearness over usual expositions, owing to the use made of the notion of "value approached"; ** and the chapters on continuous functions (v.), †† infinitesimals

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* The authors assume (p. 11) the existence and defining properties of the positive integers by a set of axioms, and show, by the example of the integer-pairs $[m, n]$, where $[mk, nk]$ (k being an integer) "is regarded as the same as $[m, k]$," and 'multiplication' and 'addition' of pairs are defined by certain equations, "that no contradiction will be introduced by adding a further axiom to the effect that besides the integers there are numbers, called fractions, such that in the extended system division is possible."

In the first place, we cannot, unless we re-define 'equality,' assert that $[m, n]=[mk, nk]$, for this is not an identity. And re-definition of 'equality' is unnecessary, for, as Frege remarked (cf. Couturat, *Les principes des mathématiques*, Paris, 1905, p. 49), mathematical equality means logical identity. Secondly, the example shows only that there is a system of pairs, which does *not* include the integers, since the pair $[m, 1]$ cannot be m (cf. this *Gazette*, May, 1906, p. 315), but includes a part of pairs in which the second term is 1, and which have a one-one correspondence with the integers, in which 'division' (which is quite different in concept to the division of integers, but was suggested by it) is always possible. The authors rightly denote the 'multiplication' and 'addition' of pairs by *new* signs. Thus the axiom of the existence of fractions *with* the integers is unjustified; while a not very great alteration affords us a definition of fractions (as classes of couples), and renders an 'axiom' unnecessary.

Much the same remarks apply to the axiomatic introduction (pp. 12-13) of negative, real, and complex numbers; while the (Huntington's) definition of a real number system by a set of postulates (pp. 13-15) is, like Hilbert's analogous attempts, subject to logical objections (cf. Couturat, *op. cit.*, pp. 40-42, 57-58; Frege, *Jahresber. d. deutsch. Math.-Ver.*, 1903).

I incline more and more to the view that, for beginners, a *geometrical* introduction of 'number' (in a wider sense) is advisable, and this seems to be suggested by our authors in the note on p. 30.

Though our authors deserve our thanks for not calling a number a *sign*, they define (p. 44) a 'variable' as a *symbol* (cf. this *Gazette*, May, 1906, p. 315).

† See Veblen, *Bull. Amer. Math. Soc.* (2), x., 1904, pp. 436-439.

‡ The *axiom* in Dedekind's work is *geometrical*; he *proves* the continuity of the real number system. But his process is equivalent to the defining of real numbers by an axiom of continuity (as is done on pp. 3-7).

§ See my paper in the *Mess. of Math.*, 1906, pp. 61-69. In this connexion, it should be remarked that the 'the' on our author's p. 33 (l. 4 from bottom) should be 'a.'

‖ *Math. Ann.*, Bd. lix., 1904, pp. 514-516.

** A member of the class of limits of $f(x)$ when x approaches a limit-point a. Since this class always has at least one member (p. 61), the existence or non-existence of *the* limit of $f(x)$ when x approaches a means, respectively, that the class has only one member or more than one member.

†† The development of the remark on the uniform continuity of $a^{\frac{m}{n}}$, and on a resulting possible definition of a^x (pp. 96-97), is especially illuminating for a student.

and infinities (vi.), derivatives and differentials (vii.),* and definite integrals (viii.),† succeed in combining rigour, modernity, comprehensibility, and interest.

The last chapter (ix.) on improper definite integrals is more advanced, and includes a treatment of the improper definite integrals, with an infinity of singularities, which were first investigated by Harnack and Hölder.

There are said to be no examples, since the book is a reference book, but many proofs and other suggestions are left to be worked out by the student.

PHILIP E. B. JOURDAIN.

Examples and Homework in Preliminary Practical Mathematics. T. I. COWLISHAW, M.A. pp. 53. Longmans. 1s.

An interleaved set of XXV. papers in practical arithmetic and algebra to quadratics, which should prove useful for evening classes. The work in arithmetic includes volumes of cones and cylinders, and in algebra graphical methods. A good feature is the early introduction of logarithms. In the examples for contracted methods needlessly long decimals (as in almost all text-books) are given. Otherwise the examples are well chosen, particularly substitutions and graphical illustrations.

Modern Commercial Arithmetic. Part I. Elementary Stage. G. H. DOUGLAS, M.A. pp. 163. Macmillan. 1s. 6d.

This book covers the ground from the Simple Rules to Profit and Loss, Interest, and Areas. The rules are clearly explained, and there is a sufficiency of good examples. A good feature is the working of questions in Proportion by the fractional method in two steps. This is probably the best way for the beginner, but it would be an improvement to replace it, as soon as the principle is grasped, by the single step. Inverse Simple Interest is separated from Direct by Compound Interest; this is surely a mistake, as all Simple Interest problems are particular cases of Proportion (in which interest, rate, time, principal, amount, respectively are to be found), whereas Compound Interest involves Exponentials. The Metric System is given at the very end of the book, too late for the beginner to get a good knowledge of it; but a good feature here is the giving of rough equivalents in English measure, *e.g.*, a hectare $=2\frac{1}{2}$ acres.

Examples in Arithmetic, with some Notes on Method. C. O. TUCKEY, M.A. pp. xii + 251 + xxxix. Bell & Sons. 3s.

These examples are sufficient for the ordinary school course in arithmetic on modern lines. Needlessly long decimals (as in all text-books) are given for approximations, *e.g.* 'find the product $728{\cdot}36152 \times {\cdot}5207436$ to four figures.' As most measures are entirely unreliable beyond three figures, no decimal in elementary work, except in money problems, should have more than four significant digits. There are several good features in the book. Square root is taken with decimals, and the right triangle and simple areas and volumes follow immediately; a clear statement is given of alterations of quantities in a given ratio (p. 41), which forms a most natural introduction to the fractional method (the best of all) of treating proportion (this is far better than Mr. Norman's method at the end of the book); and in Part II. the use of logarithmic and other tables is explained. This section would be better placed before the problems in physics. The examples would also be improved by a more frequent use of the Metric System of measures. Tables of Compound Interest, four figure logarithms, and trigonometrical ratios are given.

Clive's Arithmetics for Scheme B. Standards IV., V., VI. pp. 36, 44, 55. WM. BRIGGS, LL.D., D.C.L., M.A., B.Sc. Each 3d.

What little book work there is in these is not carefully done. For example, 'The Factors of a number are the numbers which are exactly divisible into it, *i.e.*, divisible without remainder.' This is not a sufficient definition. 16 goes into 12 without remainder $\frac{3}{4}$, but 16 is not a factor of 12. There is an abundance

* There might have been very suitably some discussion on the conception of uniform differentiability and its relations with the continuity of $f'(x)$, on p. 133. This is very necessary for an adequate appreciation of Goursat's proof of Cauchy's theorem in complex function-theory.

† The development of the theory of limits for *many*-valued functions also gives certain advantages in the treatment of the definite integral (pp. 153-154).

of suitable examples up to the unitary method and simple areas, and the metric system is very properly introduced in connection with decimals.

Blackie's New Concentric Arithmetics. Book II. D. M. Cowan. pp. 64. Blackie & Son. 3d.

This goes as far as the compound rules. It is a good little book. Decimals are introduced quite naturally with the metric system, so that familiarity with the decimal point is gained before the systematic treatment of decimals is begun. Fractions are well illustrated. "Oblong" on p. 30 should be replaced by "rectangle"; the latter word is now universally used, and is as easy to learn as the other.

Introduction to Analytic Geometry. Percy F. Smith, Ph.D., and Arthur Sullivan Gale, Ph.D. pp. viii + 217. Ginn & Co.

This introductory book to algebraic geometry is scarcely complete enough for use in English schools. It has several good points. The subject is treated by the Euclidean method of definition and theorem; this is quite the right method now that inductive treatment by graphs in elementary algebra gives the beginner a working acquaintance with the forms of the figures dealt with. The other leading feature of summarizing the steps in each demonstration is of very doubtful advantage, as it tends to make the student too dependent on his book. The area of a triangle through three points is ingeniously derived from that of a triangle with the origin as one vertex. The various forms of equation of the straight line are all well treated, the intercept form being derived from the general equation, and the perpendicular form by orthogonal projection. Coaxal circles are fully dealt with, and rotation and translation of axes serve as an introduction to the study of conics. The simplest forms of the equations are first derived from the polar equations, and all the various forms of the equation of the second degree are ingeniously and rigorously treated, the chief fault here being an insufficient discussion of the general equation of the second degree. The general ideas of tangent, normal, etc., are assisted by the treatment of the curve $y=x^3$ side by side with conics. A short account of the most interesting forms of the corresponding solids, ellipsoid, hyperboloid, etc., is given at the end of the book. The diagrams, as is usual in American books, are excellent.

Algebraic Geometry, a New Treatise on Analytical Conic Sections. W. M. Baker, M.A. pp. 325 + xxiii. Geo. Bell & Sons. 6s.

This is more than sufficient for candidates for Woolwich, for whom it is probably primarily intended, but is scarcely likely to supplant the good and more complete treatises already in use for the general student of higher mathematics. It has the excellent series of examples that we are accustomed to look for from its author, and the diagrams are throughout admirable. In the straight line, determinants might well be introduced where the general solution for two straight lines is given, and used again for the area of a triangle.

$$\Delta=\begin{vmatrix} x_1, & y_1, & 1 \\ x_2, & y_2, & 1 \\ x_3, & y_3, & 1 \end{vmatrix}$$

is by far the easiest form to remember. The equations of the straight line are well and clearly dealt with.

In the circle the equation of the polar of a point is ingeniously and satisfactorily derived from its definition as the locus of intersections of tangents from the extremities of chords through the point. The alternative treatment of conics by geometry and algebra is probably the right one, though it would certainly be better to deduce a few of the simpler general properties of conics geometrically before deriving the special equations. The methods of drawing curves from their equations are clearly given, though the work here is not rigorous. For example it is assumed that every equation of the form $(ax+by)^2+2gx+2fy+c=0$ is a parabola; whereas the converse only has been previously proved; viz., that the equation of a parabola must have the above form. A short previous treatment of the general equation would remove this blot. An excellent new feature is the use of the differential calculus in finding the equation of the tangent at a point.

Trigonometry for Beginners. REV. J. B. LOCK, M.A., and J. M. CHILD, B.A. Macmillan & Co. pp. viii + 186.

An introductory text-book to solution of triangles, without the $A \pm B$ formulae. The special feature of the book is the detailed description of instruments, including instructions for making a simple working model of a theodolite. Numerous illustrations in addition to the ordinary graphs of the ratios will greatly assist the student who has to work by himself. The difficult half angle formulae are rightly derived from the in- and e- centres. It is, however, a mistake to confine the treatment of ratios too long to acute angles, as it necessitates such dubious devices as defining $\sin(180° - A)$, $\cos(180° - A)$ as equal to $\sin A$, $-\cos A$, the idea of a negative line being first introduced on p. 161. The authors advocate the use of 5 figure tables on the ground that 4 figures are not accurate enough. As one element in the solution of a triangle is a side, which is very rarely indeed known accurately to four figures,* the contention can scarcely be maintained. The tables in the book only give (to five figures) the logs of *three* figures, so that the worker has to do his own interpolation; he would find it quicker and easier to work with seven figures from good seven figure tables.

The worked examples in the book have the rare merit of being quite sufficient without being in excess, and the book is well adapted to Stage II. of the Board of Education, for which it seems primarily intended.

Trigonometry for Beginners. J. W. MERCER, M.A. Cambridge University Press. pp. xii + 323.

This book is intended for young boys, and particularly for naval cadets. The subject is therefore treated at much greater length than is usual or necessary for ordinary classes. Only tangents, sines, and cosines are treated at the commencement; but there seems no good reason for confining these ratios to acute angles, now that boys learn the signs of lines with their graphs in algebra; the ratios present no difficulties up to 180°. Triangles are completely solved before the $A \pm B$ formulae are given, a practice which I am glad to see is extending. Examples of methods of solution are given very fully, most teachers will think needlessly so. The difficulty of the half angle formulae is cleverly got over by using a new ratio, 'the haversine of A' for $\sin^2\frac{A}{2}\left(=\frac{1-\cos A}{2}\right)$, a table being given at the end of the book. Triangles are also solved from the Traverse Table. In this case drawing on squared paper would be equally speedy and, I should expect, equally accurate. Both these tables, however, are specially useful for naval students. The tangent form given,

$$\frac{\tan\frac{A-B}{2}}{\tan\frac{A+B}{2}}=\frac{a-b}{a+b},$$

is certainly better than the old form. Co logs are used (log of reciprocal) so that all calculations are worked by addition. As the triangle is completely solved first, there seems no good reason for not deriving the $A \pm B$ formulae from the triangle formulae, as these at once give results up to $A \pm B = 180°$. The book is admirably got up, and is quite suitable for the young student for whom it is written.

E. BUDDEN.

Text-book of Mechanics. By LOUIS A. MARTIN, Jr. Vol. I. Statics. pp. xi, 142. Chapman & Hall, London, 1906. 4s.

In this book the usual propositions of elementary statics are clearly stated and adequately illustrated. The teacher who invests in Mr. Martin's book will pick up several hints as to the presentation of the subject, some of which will probably be new and acceptable. The directions as to dealing with problems involving the construction of a force triangle are particularly good, and the book gives one the impression of having been written by an author who knows from experience the troubles and difficulties of beginners. Mr. Martin's conscience has compelled

* Temperature difference between summer and winter (say 32° - 82°, F.) makes a difference of 1 in 3000 in a steel chain.

him to put in an introduction, in which mass, momentum, acceleration, absolute units and so forth are polished off in 9 pages. But the pupil's good sense may be relied upon to make him skip this, and the poundal, which is duly introduced as the unit of force at p. 6, does not make its appearance once again throughout the book. C. S. JACKSON.

CORRESPONDENCE.

The Teaching of Geometry.

I am enclosing a copy of our report on the subject of Geometry presented to the Central Association of Science and Mathematics Teachers at Chicago, Ill., which was accepted as a preliminary report, and the committee continued for work in Solid Geometry. It is to be printed as a separate document and published for distribution. In the meantime I thought you might be interested in having a copy of it to see what we are trying to accomplish. You will see that in some respects it parallels the work done by your Committee in England for the Improvement of the Teaching of Geometry; these portions had already been decided upon and outlined when I received through you the copies of the *Gazette* dealing with the subject, and they were of great encouragement to us; until that time I was unaware that anyone had recommended the omission of incommensurable cases, and I was quite pleased to find that it had been already strongly advocated. You will notice that we quote a line from your report. At Chicago our recommendations were well received, but it is hard to tell what will be the result when they are given out to the country at large. I would be glad to have the opinion of your readers on the report, either as a whole or on particular sections in which they may be interested, and also to receive the opinions of any others who would so favor us with their criticisms or support.—Yours faithfully, G. W. GREENWOOD.

Roanoke College, Salem, Va.

The Principles of Dynamics.

In his interesting paper on the Principles of Dynamics, Mr. Larden, while insisting that kinetic energy is meaningless except with reference to a system, appears to pass over the more obvious fact that force is meaningless except with reference to a frame. It seems to me that it is unnecessary and irrelevant to Dynamics to bring in the statical method of measuring forces; this plan is open to the further objection that it misleads a beginner as to the dynamical meaning of force. With regard to kinetic energy, I am afraid that I do not properly appreciate the difficulty which Mr. Larden has devoted so much space to clearing away. If K. E. is defined as $\frac{1}{2}mv^2$, with reference to C. M. of system as origin, perfectly consistent results are obtained. Considerations as to amount of heat given out appear to me to be extra-dynamical.

In connection with this subject I hope that the first writer of an elementary text-book of Dynamics which shall approach the subject logically, whoever he may be, will note the usual definition of Potential Energy, which, given as it commonly is before anything has been said about a conservative system, is totally devoid of meaning. W. D. EVANS.

The Line at Infinity.

Mr. Hardy writes (No. 61, p. 14): "Most undergraduates seem to believe that there really are points at infinity, and that they really do lie on a line, and that if you could get there you would find that $1=0$." May I say that at least one undergraduate not only believes "that there really are points at infinity," but is prepared to defend that belief? He does not mean to assert that the reality of the points at infinity is of the same kind as that of, let us say, the *Mathematical Gazette*, but he does assert that that reality of these

points is of the same kind as that of any finite point. Many people seem to find the fact that they can approximate to the position of a point by means of a dab of ink very helpful in assuring them of its real existence, but the point (2, 3) is as much an ideal construction as the famous points I and J. The undergraduate in question ventures to think that the existence of points at infinity is a logical deduction from the existence of finite points, and holding these opinions he has no difficulty in believing that these infinite points "really do lie on a line." At the same time he has no confidence that it will ever be possible to indicate their approximate positions by means of ink-spots. He has also the hardihood to express the belief that the confusion of thought consequent upon the very widespread mixing up of perceptial with conceptial space is responsible for any conjectures as to what "you would find if you could get" to the line at infinity. W. D. EVANS.

Cricket and Dynamics.

I read a little while ago in a report of a discussion at Johannesburg that Prof. Perry considered that every schoolboy ought to understand that force is the vector rate of change of momentum. I must say the idea quite astounded me. I am sure that no boy in my school would ever understand such a statement if he learned it up out of books. But a bright idea has suggested itself. If this piece of information is really so very valuable, could not the boys be taught the elementary notions of dynamics from their school games, such as cricket. Most boys care a great deal more about games than about books, but one sometimes finds a boy that likes his books and is no good at games. On the other hand, all boys have a great sense of curiosity about the why and wherefore of curious things. I believe that if dynamics could be taught by making the boys find out why they throw a cricket ball in a particular way, and why it rebounds as it does when they strike it with a bat, the athletic boys would learn something and the studious boy would begin to take an interest in his games. I can fancy the last-named boy at the end of a game of cricket surrounded by an eager crowd pumping him with questions. But unfortunately this would not help the boys with their examinations, and there is no time in class hours to try the experiment. PEDAGOGICUS.

QUERIES.

(1) Will some correspondent of the *Gazette* publish a list of the principal vulgar fractions $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots$ expressed in the forms corresponding to decimals in different scales of notation? The list need not be a long one; it might go up to $\frac{1}{12}$ and 12 as the base, or better up to $\frac{1}{20}$ and 20 as the base, and it might be conveniently put in tabular form. I do not know if such a list has been published anywhere. Might it not be worth reprinting in the *Gazette*? It has been said that the decimal system of notation is better than any other, and such a list would surely afford some test of this point. ENQUIRER.

(2) *Contracted Methods.*—Why should not the Indian method of multiplication be taught in England? It seems to get over a great many difficulties. GWILYMJI.

(3) Why is the "Italian" method of division so-called? ST. IVES.

(4) Who was the Duchayla to whom we owe a proof of the "Parallelogram of Forces" based upon the principle of the transmissibility of force?

X. Y. Z.

(5) Where can I find some account of "paradoxes" such as that of the ball suspended in a jet of air? What solutions have been suggested?

BIBIPI.

(6) It is stated that if $\alpha, \beta, \gamma, \delta$ be any quantities such that $\alpha+\beta+\gamma+\delta=0$; $a\alpha+b\beta+c\gamma+d\delta=0$, it can be *easily* shewn that

$$\Sigma(a-b)(a-c)(a-d)\alpha^2=0,$$
$$\Sigma(a-b)(a-c)(a-d)a\alpha^2=(\Sigma a^2\alpha)^2$$
$$\Sigma(a-b)^2(a-c)^2(a-d)^2\alpha^2=(\Sigma a^2\alpha)^3.$$

An *easy* proof will be welcome. FACILE DICTU.

(7) Can a triangle be constructed if we are given (1) the median from A, the bisector of B, and the perpendicular from C; (2) the feet of the three internal bisectors of its angles? TRIPLET.

(8) Can any reader of the *Gazette* give a few of the mnemonics for the first few decimal places of π? Have any been constructed for e? To how many places has e been calculated? ἐπί.

(9) Can the following be proved by *purely geometrical* means? If a, b, c, d, be the sides in order of a convex quadrilateral, and e, f its diagonals, then $2(ab+cd)(ad+bc)$ is never less than $(ac+bd+ef)(a^2+b^2+c^2+d^2-e^2-f^2)$.

GEMMI.

(10) The general equation to a parabola through $(0, 0)$, $(a, 0)$, $(0, b)$, axes rectangular, is $x^2-ax+2\lambda xy+\lambda^2(y^2-by)=0$, where λ is variable.

The envelope reduces to

$$xy\left(\frac{x}{a}+\frac{y}{b}-1\right)=0.$$

Hence, if a parabola circumscribe a triangle it is also inscribed in the same triangle.

Required an explanation of this paradox. Q.

(11) *Higher Trigonometry.* Will some "expert" reader of the *Gazette* tell me exactly *what* proofs, in what order, and with what reservations, should be adopted to cover the following ground?

(*a*) Exponential and Logarithmic Limits and Expansions.
(*b*) Convergency of Double Series.
(*c*) Expansions for $\sin m\theta$ etc.; $\sin\theta$ etc.
(*d*) Factor Formulae and extension to Partial Fractions.

Also, what form will the deduction of the expansions from $\dfrac{dy}{y}=i$ take when complete?

It will be useful if some examiner will state how much he expects from the scholarship candidate. Many promising boys drop out because they are overwhelmed by the sheer bulk of such work. Moreover, where too much is demanded, sound teaching is discouraged. CARDIFF.

(12) It does not appear that the ordinary textbook has devised any method of making clear the proofs of the relations between indices and logarithms. The proofs are often written out badly, perhaps because the method of presentation is artificial. Will some reader kindly refer me to the best method of presentation, or write a Note to the *Gazette* on the subject? X, β.

(13) Where can I find explanations of:

(1) Contour integration; (2) the simpler properties of conjugate functions; (3) the validity of integration and differentiation of infinite series—all in such a form as is readily adapted to work with a scholarship candidate who has perforce to be left much to himself? TRIP. 1892.

(14) In dealing with (Math.+Science) boys below scholarship standard, I adopted the following method for $\dfrac{d}{dx}(a^x)$.

Obtaining $\underset{h=0}{\mathrm{Lt}}\dfrac{a^h-1}{h}$ in the usual way, I got *graphs* of 2^x, 3^x, ... and showed

that, approximately, $\underset{h=0}{\mathrm{Lt}} \dfrac{a^h-1}{h}$ is constant in each case, and that for a value between 2 and 3 it might be expected, as a matter of continuity, to be unity.

What is the best assumption to make on which to base the series for a^x, and the $\underset{n=\infty}{\mathrm{Lt}} \left(1+\dfrac{1}{n}\right)^n$?

Much analysis seems sheer waste of time. METHODICUS.

NOTICE.

The Fourth International Congress of Mathematicians will be held in Rome, 6th–11th April, 1908.

Lectures on the present condition of the various branches of Mathematical Science will be delivered by Profs. Darboux, Forsyth, Hilbert, Klein, Lorentz, Mittag-Leffler, Newcomb, Picard, Poincaré, etc.

For information as to the Congress, application may be made to the General Secretary: Prof. G. Castelnuovo, 5 Piazza S. Pietro in Vincoli, Rome (Italy).

BOOKS, ETC., RECEIVED.

Leçons de Mécanique Céleste. By H. POINCARÉ. Vol. II. Part i. Pp. 165, 6 frcs. 1907. (Gauthier-Villars.)

Les Carrés Magiques. By J. RIOLLOT. Pp. iv, 120. 5 frcs. 1907. (Gauthier-Villars.)

Introduction to Infinitesimal Analysis. Functions of one Real Variable. By O. VEBLEN and N. J. LENNES. Pp. viii, 227. 2$. 1907. (Chapman and Hall.)

Elements of the Infinitesimal Calculus. By G. H. CHANDLER. Pp. vi, 319. 8s. 6d. 1907. (Chapman and Hall.)

The Teaching of Mathematics. By J. W. A. YOUNG. Pp. xviii, 351. 6s. net. 1907. (Longmans, Green.)

Elementary Geometry. By CECIL HAWKINS. With Answers. New Edition. Pp. viii, 306. 1907. (Blackie.)

Geodäsie. By A. GALLE. Pp. xi, 284. (Sammlung Schubert xxiii.) 8 m. 1907. (Göschen, Leipzig.)

Journal of the Tôkyô Mathematico-Physical Society. Vol. III. Nos. 9-11. Nov. 1906—Jan. 1907.

The American Journal of Mathematics. Edited by F. MORLEY and others. Jan. 1907. Vol. XXIX. No. 1. $ 1.50. (Johns Hopkins Press, Baltimore.)

The Groups which contain less than Fifteen Operators of Order Two. G. A. MILLER. *Concerning the Improper Definite Integral.* N. J. LENNES. *On the Congruence of Axes in a Bundle of Linear Line Complexes.* O. P. AKERS. *On Septic Scrolls having a Rectilinear Directrix.* C. H. SISAM.

Revista Trimestral de Matematicas. Edited by J. R. y CASAS. No. 21. (Zaragoza.)

Wiskundig Tijdschrift. Edited by F. J. VAES and others. April, 1907. (Blom, Culemborg.)

Murray's School Arithmetic. By A. J. PRESSLAND. Pp. vii, 207, xl. 1907. With Answers. 2s. 6d. Without Answers. 2s. (Murray.)

Supplementary Exercises to Murray's School Arithmetic. By A. J. PRESSLAND. Pp. 84. 6d. 1907. (Murray.)

Theoretical Mechanics. By J. H. JEANS. Pp. v, 364. 1907. (Ginn.)

Vorlesungen über Geschichte der Mathematik. By MORITZ CANTER. Vol. I. 3rd Edition. Pp. vi, 941. 24 m. 1907. (Teubner.)

Lehrbuch der Elastizität. By A. E. H. LOVE. Pp. xvi, 664. 16 m. 1907. (Teubner.)

Abhandlungen zur Geschichte der Mathematischen Wissenschaften, xxii. *Briefwechsel zwischen C. G. J. Jacobi und M. H. Jacobi.* Pp. xx, 282. 6.90 m. 1907. (Teubner.)

Lehrbuch der Funktionentheorie. By W. F. OSGOOD. Vol. I. Part ii. Pp. xii, 307-642. 7.60 m. 1907. (Teubner.)

GLASGOW: PRINTED AT THE UNIVERSITY PRESS BY ROBERT MACLEHOSE AND CO. LTD.

... it is an extraordinary Algebraist, but I expect ...
... Art from his hand; But (If I might have my desire) I would rather ...
himself to the cultivating of Pure Geometry. That is a large Subject, worthy ...
of a Professor, and is abundantly more entertaining than the Contemplation of mere
abstract quantities, which are the proper objects of Algebra; but that, properly speaking
is but an introduction to Mathematicks, as Logick is to Philosophy. And it is my opi-
nion that the prevailing humour of treating Geometry so much in an Algebraical way,
has prevented many Noble discoveries, that might otherwise have been made, by follow-
ing the Methods of the Ancient Geometricians. But these things are now out of my way.

I do here send you your tickett. I have been very diligent in looking after your
luck, but dont find it yet determined, and the hopes of good Fortune seem to be increased
by the drawing. My wishes for your success cannot be greater than they are; and (if the
expectations I have of a benefit ticket in another way dont prove vain) I shall still wish
you the 12000, tho' you banter me for giving you the preference to my self. I have
set bounds to my desires, of the goods of this world, and have a very fair prospect of
their being more than satisfied, without a prize in the Lottery. I have but one reason
to desire that my Fortunes might be greater, than they are likely to be, That I might be
able to divide them with my Friends who deserve them better. The greatest pleasure,
that I can conceive in this World, you have the enjoyment of, In seeing the most usefull
learning for the good of mankind flourish, & your self contributing very much to it.
All the riches in the world cant give any satisfaction like that of doing good. I wish I
could come and make you a visit, but I cant yet tell when I may propose to my self
that happiness, for my little affairs are now in a very odd posture, that makes it im-
... judge what I shall be likely ... You see I talk enigmatically, but yo...
...clusions from it.

Pray give my humble service to all friends in Coll: and continue to
me with the greatest assurance

Bishop's head in St Paul's Church yard
24 Nov: 1711

Dear Tom

Yr most sincere friend and
humble servant
Brook Taylor

Tell Mr Bowtell I shall expect him
with impatience on Tuesday night.

See page 97.